UNIT 6 – SIMILARITY OF FIGURES

Assignment	Title	Work to complete	Complete
1	***Review – Proportional Reasoning***	Cross Multiply and Divide	
2	***Similar Figures***	Similar Figures	
3	***Determining Sides in Similar Figures***	Determining Sides in Similar Figures	
4	***Determining Angles in Similar Figures***	Determining Angles in Similar Figures	
	Quiz 1		
5	***Scale Factor in Similar Figures***	Scale Factor in Similar Figures	
6	***Calculating Scale Factor***	Calculating Scale Factor	
7	***More Scale Factor***	More Scale Factor	
	Quiz 2		
8	***Working with Similar Figures***	Working With Similar Figures	
9	***Drawing Similar Figures***	Drawing Similar Figures	
10	***Similar Triangles***	Similar Triangles	
Practice Test	**Practice Test** How are you doing?	Get this page from your teacher	
Self-Assessment	**Self-Assessment** ***“Traffic Lights”***	On the next page, complete the self-assessment assignment.	
Chapter Test	**Chapter Test** Show me your stuff!		
Mental Math	**Mental Math** Non-calculator practice		

Traffic Lights

In the following chart, show how confident you feel about each statement by drawing one of the following: ☺, 😐, or ☹. Then discuss this with your teacher ***BEFORE*** you write the test!

Statement	☺ 😐 ☹
After completing this chapter;	
• I can determine if polygons are similar by their corresponding angle measures	
• I can determine if polygons are similar by their corresponding side lengths	
• I can explain why two polygons are not similar	
• I can find the scale factor between the corresponding sides of similar polygons	
• I can draw a polygon that is similar to another polygon	
• I can explain why two right angle triangles with one shared acute angle are similar	

Vocabulary: Unit 6

congruent
corresponding angles
corresponding sides
equilateral triangle
isosceles triangle
proportion
ratio
scale factor
similar figures

REVIEW – PROPORTIONAL REASONING

A **ratio** is a comparison between two numbers measured in the same units.

A ratio can be expressed in three ways as shown below:

as a fraction $\frac{9}{16}$

in words by using the word "**to**" **9 to 16**

a notation using colon **: 9 : 16**

Ratios, like fractions, can be simplified. For example, the ratio **150 : 15** can also be expressed

$$\frac{150}{15}$$

which can be simplified $\frac{150 \div 15}{15 \div 15} = \frac{10}{1}$

Notice that the numerator of the fraction is larger than the denominator. This can be common with ratios.

If two ratios are equivalent (equal), the first (top) term of each ratio compares to the second (bottom) term in an identical manner. You can represent this equivalence in the two ratios here:

$$\frac{150}{15} = \frac{10}{1}$$

An equation showing equivalent ratios is called a **proportion**.

Cross Multiply and Divide

When two fractions are equal to each other, any unknown numerator or denominator can be found. The following example shows the process.

Example 1: Find x when $\frac{x}{3} = \frac{2.1}{4}$

Solution: Cross multiply means multiply the numbers across the equals sign (the arrow). The divide part means divide that result by the number opposite the unknown (x) as shown below.

$$\frac{x}{3} \nearrow \frac{2.1}{4}$$

This gives the result $x = 3 \times 2.1 \div 4$

In other words, if $\frac{x}{3} = \frac{2.1}{4}$, then $x = 3 \times 2.1 \div 4 = 1.575$

It does not matter where the unknown (x) is in the proportion, This process works for all situations.

This process can also be used when one side of the equal sign is not in fraction form.

Example 2: Find x when $27 = \frac{x}{3}$

Solution:

Step 1. The number 27 is the same as $\frac{27}{1}$. So, place a 1 under the 27 to get:

$$\frac{27}{1} = \frac{x}{3}$$

Step 2. Cross multiply and divide as above $\frac{27}{1} = \frac{x}{3}$ to solve.

So: $x = 27 \times 3 \div 1$
$x = 81$

ASSIGNMENT 1 – CROSS MULTIPLY AND DIVIDE

Find the missing term by using cross multiply and divide. If necessary, round answers to one decimal place. SHOW YOUR WORK.

1) $\frac{x}{7} = \frac{4}{35}$

2) $\frac{2}{9} = \frac{x}{27}$

3) $\frac{3}{18} = \frac{25}{x}$

4) $\frac{3.2}{x} = \frac{16}{4}$

5) $\frac{x}{6} = \frac{0.5}{17}$

6) $\frac{25}{x} = \frac{40}{200}$

SIMILAR FIGURES

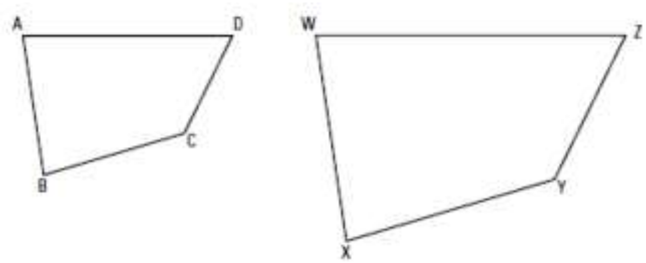
Two figures are said to be **similar figures** if they have the same shape but are different sizes. A diagram drawn to scale to another diagram makes two similar figures. Also, an enlargement or a reduction of a photograph when reproduced to scale, produces similar figures.

Corresponding angles are two angles that occupy the same relative position on similar figures. **Corresponding sides** are two sides that occupy the same relative position in similar figures. When we use the term "relative position," you must remember that the one figure might be turned compared to the other figure. It is necessary to look arrange the two figures so they look the same before deciding which angles or sides correspond.

> The key points for two figures to be similar are:
> - corresponding angles must be the equal
> - corresponding sides must be in the same proportion.

When labelling figures, strings of capital letters in alphabetical order are used. The order of the letters tells you which sides and angles correspond.

Example 1: The quadrilaterals ABCD and WXYZ are similar. State the corresponding sides and angles.



Solution:

$\angle A = \angle W$

$\angle B = \angle X$

$\angle C = \angle Y$

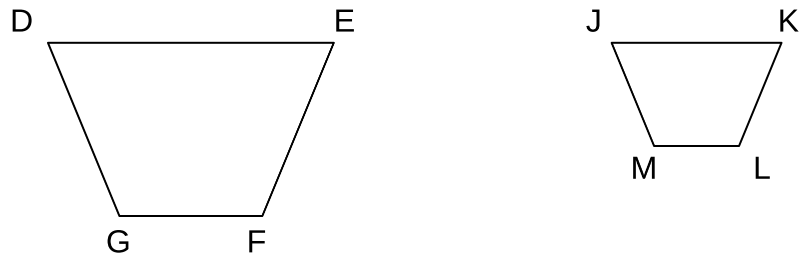
$\angle D = \angle Z$

$$\frac{AB}{WX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{DA}{ZW}$$

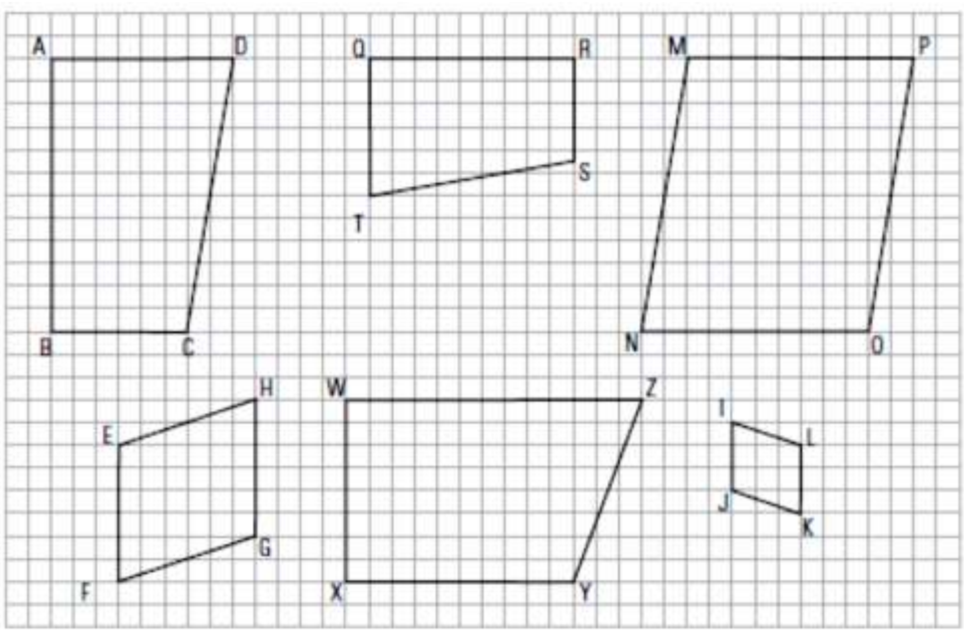
The two quadrilaterals are similar. Because ABCS is similar to WXYZ, we can use a symbol "~" which means "is similar to." So ABCD ~ WXYZ

ASSIGNMENT 2 – SIMILAR FIGURES

1) Trapezoid DEFG is similar to trapezoid JKLM, as shown below. State the corresponding sides and angles.



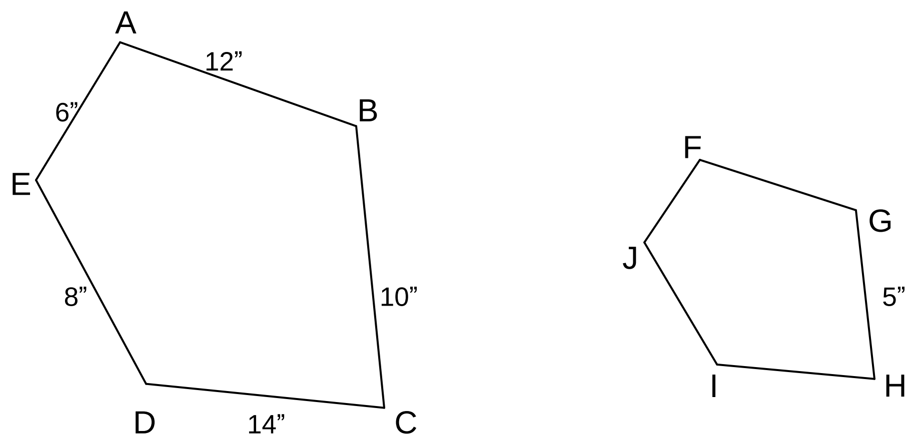
2) Identify two pairs of similar polygons below by letter names. Explain your reasoning.



DETERMINING SIDES IN SIMILAR FIGURES

When working with the length of sides in similar figures, because the figures are always a reduction of enlargement of each other, the ratio of the corresponding sides is always the same. What this means is that by using a proportion, you can determine the lengths of all the sides in both figures.

Example 1: The two figures below are similar. Find the lengths of the side of the smaller figure.



Solution: Use a proportion to solve each side in the smaller figure.

Set up proportions using BC and GH as those two sides define the ratio. For this example, make sure the sides from the big figure are always on the top and the sides for the small figure are always on the bottom.

$$\frac{BC}{GH} = \frac{AB}{FG} = \frac{10}{5} = \frac{12}{FG}$$ FG = 5 × 12 ÷ 10 = 6 inches

Using the same procedure:

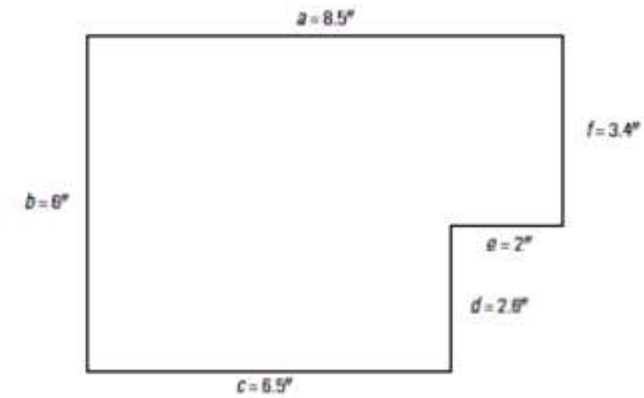
$$\frac{BC}{GH} = \frac{CD}{HI} = \frac{10}{5} = \frac{14}{HI}$$ HI = 5 × 14 ÷ 10 = 7 inches

$$\frac{BC}{GH} = \frac{DE}{IJ} = \frac{10}{5} = \frac{8}{IJ}$$ IJ = 5 × 8 ÷ 10 = 4 inches

$$\frac{BC}{GH} = \frac{EA}{JF} = \frac{10}{5} = \frac{6}{JF}$$ JF = 5 × 6 ÷ 10 = 3 inches

The lengths of the smaller figure are: FG = 6 in., HI = 7 in., IJ = 4 in. And JF = 3 in.

Example 2: Tara has made a diagram of her bedroom. On the diagram, the walls have the following lengths:



The longest wall is actually 12.75 feet. What are the actual lengths of the other 5 walls?

Solution: Set up a proportion using abbreviations for the diagram walls ("d") and the actual walls ("a") as well as the numbers. Use x as the unknown length.

Start with wall *a* (the longest) and wall *b*.

$\frac{d}{a} \qquad \frac{8.5}{12.75} = \frac{6}{x} \qquad x = 6 \times 12.75 \div 8.5 = 9$

Because the actual wall is in feet, the actual length of wall *b* is 9 feet.

Use the same procedure to find the length of the other walls.

Wall *a* (the longest) and wall *c*:

$\frac{d}{a} \qquad \frac{8.5}{12.75} = \frac{6.5}{x} \qquad x = 6.5 \times 12.75 \div 8.5 = 9$ The actual length of wall *c* is 9.75 feet.

Wall *a* (the longest) and wall *d:*

$\frac{d}{a} \qquad \frac{8.5}{12.75} = \frac{2.6}{x} \qquad x = 2.6 \times 12.75 \div 8.5 = 3.9$ The actual length of wall *d* is 3.9 feet.

Wall *a* (the longest) and wall *e:*

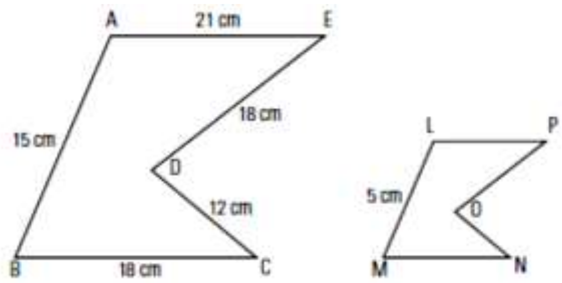
$\frac{d}{a} \qquad \frac{8.5}{12.75} = \frac{2}{x} \qquad x = 2 \times 12.75 \div 8.5 = 3$ The actual length of wall *e* is 3 feet.

Wall *a* (the longest) and wall *f:*

$\frac{d}{a} \qquad \frac{8.5}{12.75} = \frac{3.4}{x} \qquad x = 3.4 \times 12.75 \div 8.5 = 5.1$ The actual length of wall *f* is 5.1 feet.

ASSIGNMENT 3 – DETERMINING SIDES IN SIMILAR FIGURES

1) The two figures below are similar. Find the lengths of the sides in the smaller figure.

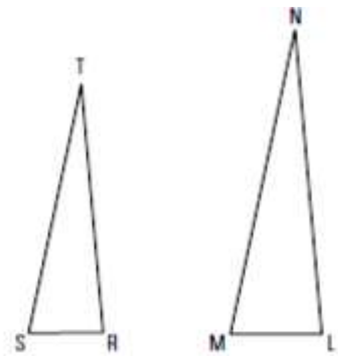


2) On a blueprint, a room measures 2.75 inches by 1.5 inches. If 1 inch represents 8 feet, what are the actual dimensions of the room? Hint: set up two proportions, one for each dimension.

DETERMINING ANGLES IN SIMILAR FIGURES

Since corresponding angles in similar figures must be equal, the only difficulty with determining the angle measures is making sure that the figures are arranged so they look the same. Sometimes this will already be done for you. But other times, you must carefully look at this arrangement.

Example: If ΔRST is similar to ΔLMN, and the angle measure for ΔLMN are as listed below, what are the angle measure for the angles in ΔRST?



$\angle$ L = 85^0

$\angle$ M = 78^0

$\angle$ N = 17^0

Solution: Determine which angles correspond, and those angle measures are equal.

Because of the naming of the triangles, we know that:

$\angle$ L = $\angle$ R = 85^0

$\angle$ M = $\angle$ S = 78^0

$\angle$ N = $\angle$ T = 17^0

ASSIGNMENT 4 – DETERMINING ANGLES IN SIMILAR FIGURES

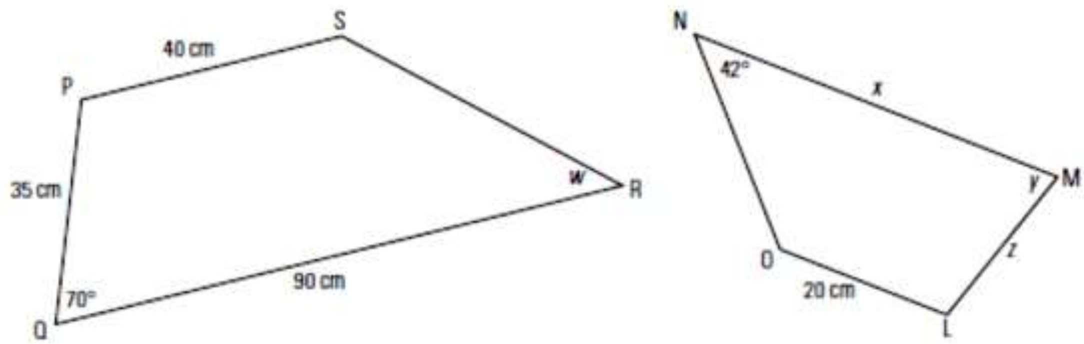
1) Two figures ABCDEF and GHIJKL are similar. The angle measures below are given. State the corresponding angles and their measures.

$\angle$ J = 73^0

$\angle$ B = 21^0

$\angle$ K = 40^0

2) If trapezoid PQRS is similar to LMNO, what are the values of *w*, *x*, *y*, and *z*? Show all your calculations and reasoning.



3) A pentagon has interior angles of 108^0, 204^0, 63^0, 120^0, and 45^0. Rudy wants to draw a similar pentagon with sides twice as long as the original. What size will the angles be?

<u>ASK YOUR TEACHER FOR UNIT QUIZ 1.</u>

<u>SCALE FACTOR IN SIMILAR FIGURES</u>

When figures are enlarged or reduced, this is often done by a scale factor. A **<u>scale factor</u>** is the ratio of a side in one figure compared to the corresponding side in the other figure. Earlier in this unit, we used the ratio of two corresponding sides in a proportion to calculate other sides. The difference with using a scale factor is the ratio when using scale factor is that it is always compared to 1. So a proportion is not necessary when the scale factor is 1: some number, e.g. 1:500.

Usually the scale factor is a single number: example, the scale factor is 1.5 or the scale factor is one quarter. Whether dealing with an enlargement or a reduction, the process of solving the problem is the same. To solve this, multiply the original lengths by the scale factor to produce the scaled lengths.

<u>Example 1</u>: A tissue has the dimensions of 9 cm by 10 cm. The company that makes the tissues wants to increase the dimensions of the tissues by 1.7. What are the new dimensions of the tissues?

<u>Solution</u>: To get the new size, multiply each dimension by 1.7.

length: 10 cm × 1.7 = 17 cm

width: 9 cm × 1.7 = 15.3 cm

Scale factors are also used on maps where a unit on a map represents a certain actual distance on the ground. For example, a scale factor might be 1 cm represents 5 km.

<u>Example 2</u>: The scale on a neighbourhood map shows that 1 cm on the map represents an actual distance of 2.5 km.

a) On the map, Waltham Street has a length of 14 cm. What would the actual length of street be?

b) Centre Street has an actual length of 25 km. What would the length of the street be on the map?

<u>Solution</u>:

a) Multiply the map length by the scale factor.

14 cm × 2.5 = 35 km

b) Divide the actual distance by the scale factor.

25 km ÷ 2.5 = 10 cm

Proportions can also be used, including the English words and numbers, as before.

ASSIGNMENT 5 – SCALE FACTOR IN SIMILAR FIGURES

1) The scale on a map is 1 cm: 500 m.

a) What distance is represented by a 12.5 cm segment on the map?

b) How long would a segment on the map be if it represented 1.5 km?

2) Teresa is making origami boxes by folding paper. The first box is 12 cm by 8 cm by 4 cm. If the next box is scaled down to ¼ of the previous box, what are the dimensions of the new box?

3) Scott was asked to scale a drawing to 75%. If one side in the drawing was 15 cm, what was the size of the new drawing?

4) Jason wants to build a model of his house. He is using a scale factor of 1 cm represents 3 m in actual size. If one room in his house is 6.5 m by 4.8 m by 2.8 m, what will the dimensions of the model be, in centimetres?

5) A sporting goods store has a miniature tent on display. The regular 6 person tent is 12 feet long and 10 feet wide. The 6 person tent has been reduced to a factor of 8 to make the miniature tent. What are the dimensions of the miniature tent?

CALCULATING SCALE FACTOR

In the previous section, we used a given scale factor to calculate the length of sides when a figure is enlarged or reduced. In this section, we will learn about calculating the scale factor when the two corresponding sides in similar figures are given.

Use a proportion to determine the scale factor. Remember, a scale factor is always 1:x where x is the number we are looking for. It may be stated as just a number, but it is really a ratio.

Example 1: Adam is drawing a scale drawing of a staircase. On the drawing, the height of one stair is 0.5 cm while the actual height of the stair is 20 cm. What was the scale factor that Adam used?

Solution:
Set up a ratio and divide to calculate the scale factor.

$$\frac{\text{drawing}}{\text{actual}} \qquad \frac{0.5}{20} = \frac{1}{x}$$

Scale Factor = x = 20 × 1 ÷ 0.5 = 40

It is also important to note that when calculating scale factor, the units of the two numbers MUST be the same. You cannot calculate scale factor with cm and metres, for example. You must change one unit into the other before using the proportion.

Example 2: Tara drew a diagram of her bedroom. In the diagram, the longest wall is 8.5 inches, but it actually measures 12.75 feet. What scale factor did Tara use when she made the diagram?

Solution: Convert the units all to inches and then set up a proportion.

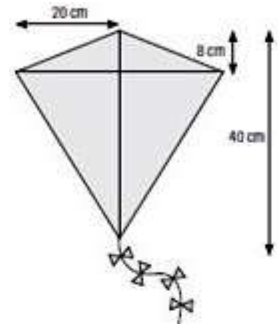
Remember: 1 foot = 12 inches
So, 12.75 feet × 12 inches = 153 inches

$$\frac{\text{drawing}}{\text{actual}} \qquad \frac{8.5}{153} = \frac{1}{x}$$

Scale Factor = x = 153 × 1 ÷ 8.5 = 18

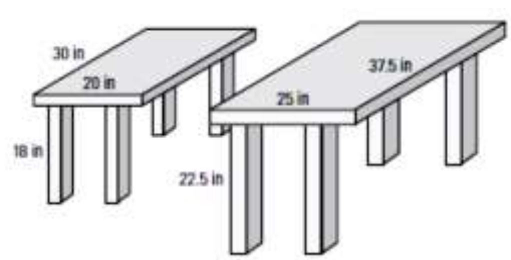
ASSIGNMENT 6 – CALCULATING SCALE FACTOR

1) Kira made the kite shown below, but decided she wanted to make a second one that was bigger. Her second kite has a tail that is 49 cm long. What scale factor did Kira use to make the second kite?



tail length = 28 cm

2) Simrin has built two tables. The second table is a slightly larger version of the first. Using the dimensions below, calculate the scale factor Simrin used to make the second table.



3) David's house is 55 feet wide. A drawing of his property shows the house 10 in wide. What is the scale factor used in the drawing?

MORE SCALE FACTOR

Not all scale factors you will be given are in the form 1:x. Often, the 1 will be some other number. When this is the case, use a proportion to solve the problem.

Example 1: Jacob is building a model of a room using a scale factor of 6:200. If the dimensions of the room are 650 cm by 480 cm, what will the dimensions of the model be?

Solution: Set up a proportion and solve. One proportion for each dimension is necessary.

$$\frac{\text{model}}{\text{actual}} \quad \frac{6}{200} = \frac{x}{650} \qquad x = 6 \times 650 \div 200 = 19.5 \text{ cm}$$

$$\frac{\text{model}}{\text{actual}} \quad \frac{6}{200} = \frac{x}{480} \qquad x = 6 \times 480 \div 200 = 14.4 \text{ cm}$$

The dimensions of the model are 19.5 cm by 14.4 cm.

Example 2: The scale of a photograph of an organism under a microscope is 75:2. If the photograph has a dimension of 30 mm, how long was the original organism?

Solution: Set up a proportion and solve.

$$\frac{\text{photograph}}{\text{actual}} \quad \frac{75}{2} = \frac{30}{x} \qquad x = 2 \times 30 \div 75 = 0.8 \text{ mm}$$

The original organism was 0.8 mm long.

ASSIGNMENT 7 – MORE SCALE FACTOR

1) The scale of a model airplane to the actual airplane is 2:45. If the model is 38 cm long, how long is the actual plane?

2) The scale of a model to its original is 3:5. If the original is 75 cm, what is the size of the model?

3) Ioana made this Ukrainian embroidery pattern for a dance costume. She wants to reduce the pattern with a scale factor of 3:10. What will the new length and width be?

3 cm

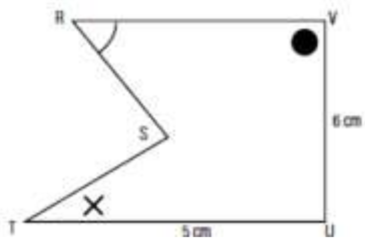
6 cm

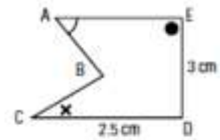
ASK YOUR TEACHER FOR UNIT QUIZ 2.

WORKING WITH SIMILAR FIGURES

In the first part of this unit, you learned about similar figures and how to find their corresponding sides and angles. In this section you will determine if two figures are similar, and what changes you can make to a shape to keep it similar to the original.

<u>Example 1</u>: looking at the two figures below, are they similar? If so, explain how you know. If not, explain what is missing or wrong. The angles marked with the same symbol are equal.





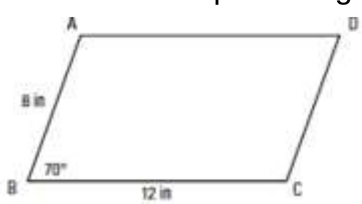
<u>Solution</u>:

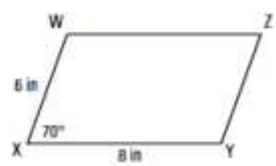
You can see that 3 of the angles in the large figure are equal to their corresponding angles in the smaller figure.

$\angle R = \angle A \quad \angle T = \angle C \quad \angle V = \angle E$

But you cannot state that the other 2 pairs of corresponding angles are equal as there is no evidence to support that. Therefore, you cannot state that the 2 figures are similar.

<u>Example 2</u>: Determine if the two parallelograms below, ABCD and WXYZ, are similar.





<u>Solution</u>:

Facts about parallelograms: 1) opposite angles are equal
2) interior angles always add up to 360^0.

So, $\angle A = \angle C \quad \angle B = \angle D = 70^0$
$\angle X = \angle Z \quad \angle X = \angle Z = 70^0$
Because the 70^0 angles correspond, the other angles must also correspond.
So, $\angle A = \angle C = \angle W = \angle Y$ and all corresponding angles are equal.

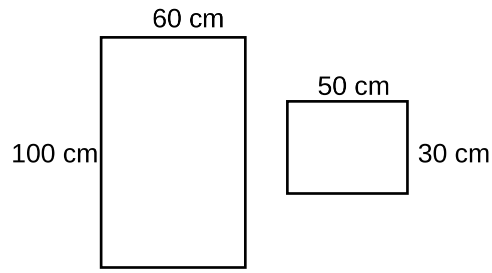
For the parallelograms to be similar, the sides would have to be proportional:

$\frac{AB}{BC} = \frac{WX}{XY} \qquad \frac{AB}{BC} = \frac{8}{12} \qquad \frac{WX}{XY} = \frac{6}{8}$

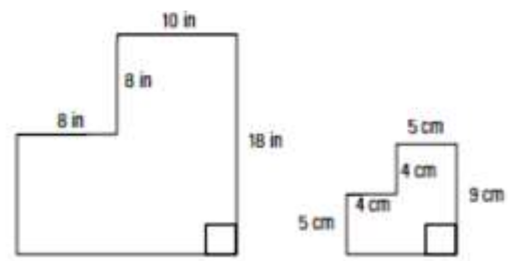
But $\frac{8}{12} \neq \frac{6}{8}$ So the sides are <u>not proportional</u> and the figures are **<u>not</u>** similar.

ASSIGNMENT 8 – WORKING WITH SIMILAR FIGURES

1) Brad says that the two rectangles below are not similar because $\frac{60}{50}$ does not equal $\frac{100}{30}$. Is Brad right? Explain.



2) Colin says that the two figures shown below are similar, but Elsie disagrees. Elsie says that they don't have enough information to determine if the figures are similar. Who is right? Show your calculation.



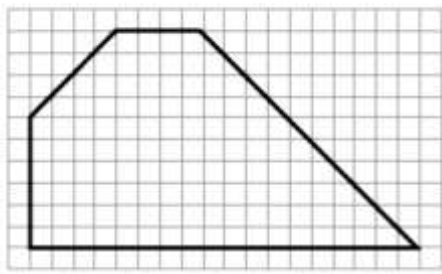
3) Aiden frames a photo that is 24 inches by 36 inches with a 4 inch frame. Is the framed photo similar to the unframed photo? Show your calculations.

4) Jeremy saw three different sized door mats at the store. They measured 36 in by 28 inches, 27 inches by 21 inches, and 24 inches by 18 inches. Are the three mats similar?

DRAWING SIMILAR FIGURES

Artists, architects, and planners use scale drawings in their work. The diagrams or models should be in proportion to the actual objects so that others can visualize what the real objects look like accurately.

Example: Use graph paper to draw a figure similar to the one given, with the sides 1.5 times the length of the original. Remember that the corresponding angles must be equal.

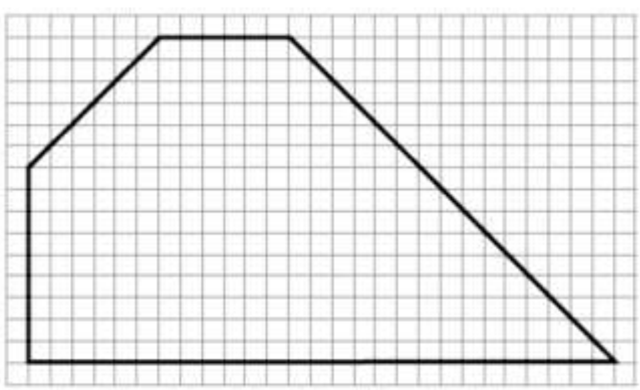


Solution: Determine the lengths of the sides by counting the squares on the grid paper. Then multiply those lengths by 1.5 to get the lengths of the new, similar figure. Draw it on the grid paper.

The lengths of the sides, starting in the bottom left corner and going clockwise around the figure are: 6 squares, 4 diagonals, 4 squares, 10 diagonals, 18 squares.

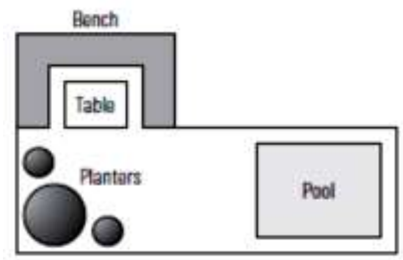
The new lengths are:

$6 \times 1.5 = 9$ squares
$4 \times 1.5 = 6$ diagonals
$4 \times 1.5 = 6$ squares
$10 \times 1.5 = 15$ diagonals
$18 \times 1.5 = 27$ squares

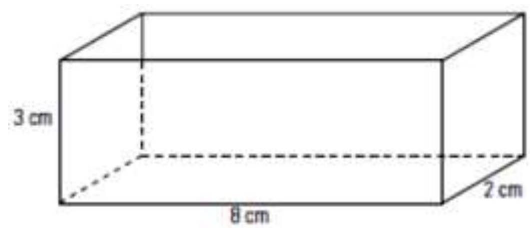


ASSIGNMENT 9 – DRAWING SIMILAR FIGURES

1) Ralph has drawn the plan below for his backyard. But he finds that it is too small to fit all the details on. Redraw the diagram at 2.5 times the size of the original.



2) Draw a rectangular prism similar to the one below with the sides ½ the length of the original.



SIMILAR TRIANGLES

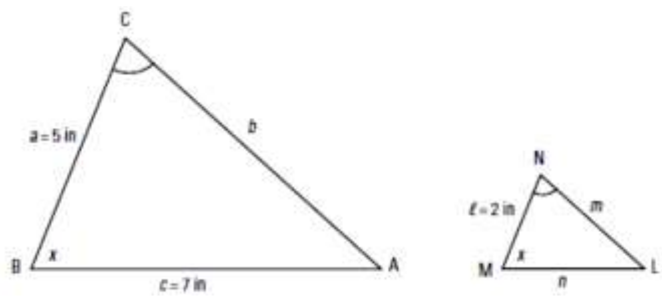
Similar triangles are very useful in making calculations and determining measurements. There are certain things to know about triangles before proceeding. Triangles always have 3 sides and three angles. The sum of the angles of a triangle is always 180^0.

If two corresponding angles are equal, the third angles will also be equal because the sum must be 180^0.

There are several special triangles – an **isosceles triangle** has 2 sides equal in length, and the two angles opposite these sides are of equal measure. An **equilateral triangle** has all three sides equal in length and all three angles equal in measure to 60^0.

Two triangles are similar if any two of the three corresponding angles are **congruent**, or one pair of corresponding angles is congruent and the corresponding sides beside the angles are proportional. Congruent means the same in size and shape.

Example 1: Given the two triangles below, find the length of *n*.



Solution: Confirm that the triangles are similar, and then use a proportion to solve for *n*.

From the markings in both triangles, you know that the two of the three angles are congruent.

$\angle C = \angle N$ and $\angle B = \angle M$

Therefore, triangles are similar and we can state $\Delta ABC \sim \Delta LMN$

To solve for *n*, set up a proportion and solve.

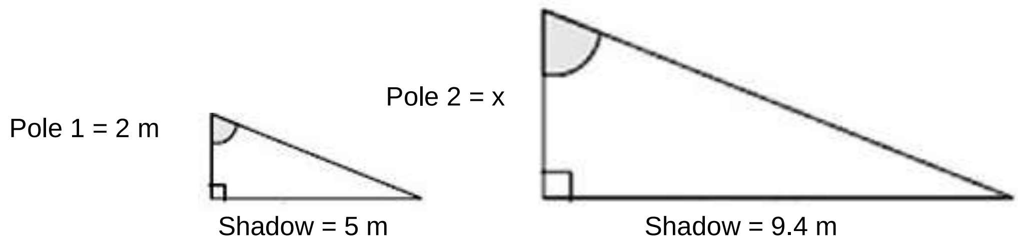
$\frac{a}{\ell} = \frac{c}{n}$ $\frac{5}{2} = \frac{7}{n}$

$n = 7 \times 2 \div 5 = 2.8$

Side *n* is 2.8 in long.

Example 2: Kevin notices that a 2 m pole casts a shadow of 5 m, and a second pole casts a shadow of 9.4 m. How tall is the second pole?

Solution: First, always make a diagram if one is not provided. Then confirm that the triangles are similar, and then use a proportion to solve for x.



Notice that 2 of the three corresponding angles are congruent. The third angles are also equal because the angle between the rays of the sun and the poles is the same in both cases. So the triangles are similar.

Now set up a proportion to solve for x.

$$\frac{\text{height of pole 1}}{\text{shadow 1}} = \frac{\text{height of pole 2}}{\text{shadow 2}}$$

$$\frac{2}{5} = \frac{x}{9.4}$$

$$x = 9.4 \times 2 \div 5 = 3.8$$

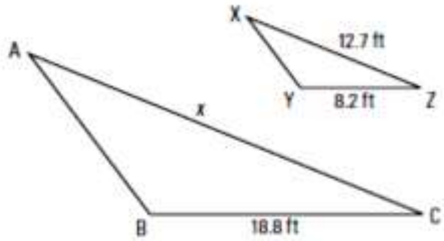
$$x = 3.8 \text{ m}$$

The height of pole 2 is 3.8 m tall.

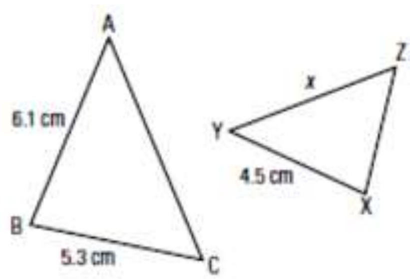
ASSIGNMENT 10 – SIMILAR TRIANGLES

1) In each of the following diagrams, **ΔABC ~ ΔXYZ**. Find the length of the indicated sides, to one decimal place. Watch the arrangement of the triangles carefully!

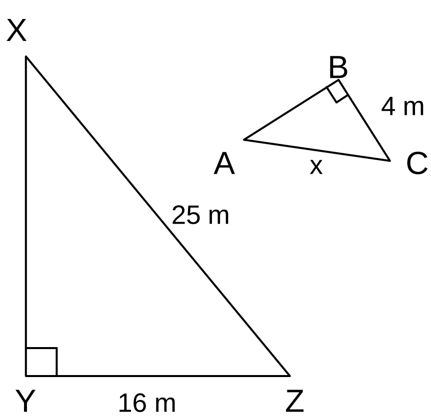
a)



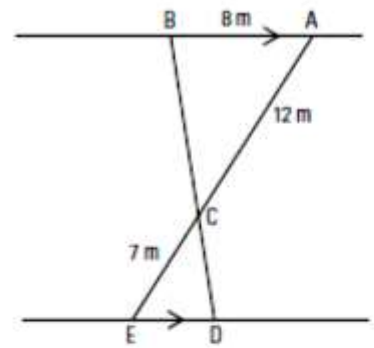
b)



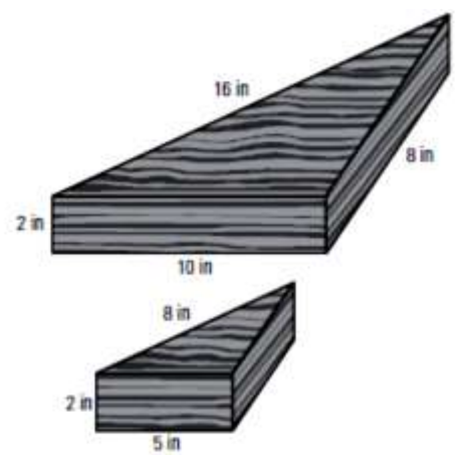
c)



2) In the following diagram, AB is parallel to ED, AB = 8 m, AC = 12 m, and CE = 7 m. What is the length of ED, to one decimal place?



3) Sean has cut two triangular shapes from a block of wood, as shown below. Are the two faces of the blocks similar? Are the two blocks similar?



4) Julian is visiting the Manitoba Legislative Building in Winnipeg. He sees a statue of Louis Riel. Use the information from the diagram below to determine the height of the statue (without the base).