<u>AWM 11 – UNIT 4 – TRIGONOMETRY OF RIGHT TRIANGLES</u>

Assignment	Title	Work to complete	Complete
1	***Triangle Review***	Pythagorean Theorem Practice	
2	***Trigonometry Review***	Trigonometry	
3	***The Trigonometric Ratios:*** ***The Sine Ratio*** ***The Cosine Ratio*** ***The Tangent Ratio***	The Trigonometric Ratios	
4	***Using The Sine Ratio*** ***Using The Cosine Ratio*** ***Using The Tangent Ratio***	Finding Sides In Right Triangles	
	Quiz 1		
5	***Angle of Elevation and Depression***	Angle of Elevation and Depression	
6	***The Trigonometric Ratios***	Word Problems	
7	***Finding Angles in Right Triangles***	Finding Angles in Right Triangles	
	Quiz 2		
8	***Solving Complex Problems***	Working with Two or More Triangles	
9	***Solving Complex Problems in the Real World***	Working with Triangles in 3-D	
Practice Test	**Practice Test** How are you doing?	Get this page from your teacher	
Self-Assessment	**Self-Assessment** ***"Traffic Lights"***	On the next page, complete the self-assessment assignment.	
Chapter Test	**Chapter Test** Show me your stuff!		

Traffic Lights

In the following chart, show how confident you feel about each statement by drawing one of the following: ☺, 😐, or ☹. Then discuss this with your teacher ***BEFORE*** you write the test!

Statement After completing this chapter;	☺ 😐 ☹
• I can use the Pythagorean theorem to calculate the missing side of a right triangle	
• I know when to choose sine (sin), cosine (cos) or tangent(tan) based on the information given	
• I can use the three basic trigonometric functions (sin, cos, tan) to find a missing side or angle of a right triangle	
• I can use the three basic trigonometric ratios to solve problems involving two or three triangles	
• I can determine the angle of elevation and the angle of depression for words or a diagram, and use them with the trigonometric ratios	
• I can use the three basic trigonometric ratios to solve problems in both 2-D and 3-D contexts	
• I can determine if the solutions that I find are reasonable based on my knowledge of the lengths of the sides in a triangle	

Vocabulary: Unit 4

angle of depression
angle of elevation
cosine
hypotenuse
leg
right triangle
sine
tangent

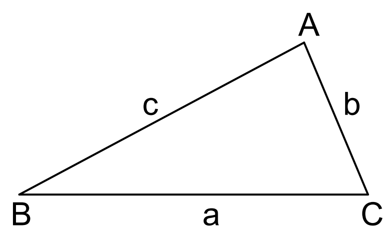
TRIANGLE REVIEW

In this unit, you will be looking at triangles, specifically right angle triangles, also called right triangles. You will review the basic trigonometric ratios and the Pythagorean Theorem, and then learn to apply these situations that have 2 or 3 triangles. But first it is necessary to review some facts about triangles.

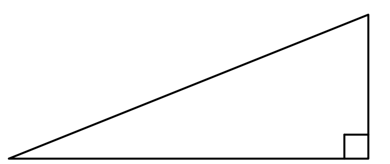
Fact 1: Every triangle contains 3 sides and 3 angles or vertices (plural of vertex).

Fact 2: The measurements of these angles always total 180^0.

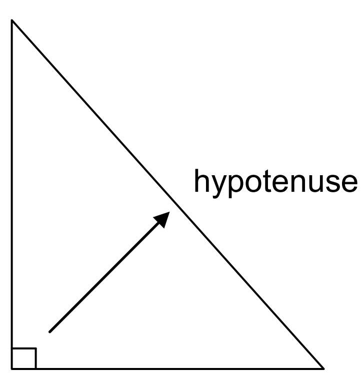
Fact 3: To identify the side or vertex in a triangle, it is important to label the triangle following a standard routine. Each vertex of a triangle is labeled with a capital case letter – like “A” - and each side is labeled with the lower case letter that matches the opposite vertex. An example is below.



Fact 4: A triangle that contains a 90^0 angle (a right angle) is called a right triangle (or right-angle triangle). ALL triangles in this unit will be right triangles.



Fact 5: The side of the triangle that is opposite the 90^0 angle is always called the **hypotenuse**. It is labelled in the triangle below. The other two sides of the triangle are called legs.

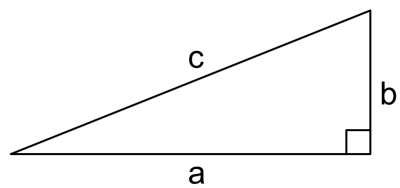


Fact 6: The hypotenuse is **always** the longest side in the triangle. It is always opposite the largest angle which is the 90^0 or right angle.

Fact 7: Pythagorean Theorem states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse. So in ΔABC with the right angle at C, the following relationship is true:

$$c^2 = a^2 + b^2$$

where a and b are the other 2 legs of the triangle.



We can also rearrange the equation to find the length one of the legs;

$$c^2 = a^2 + b^2$$

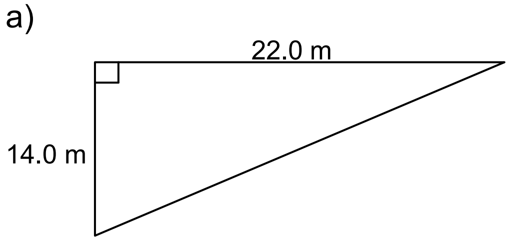
$$a^2 = c^2 - b^2$$

$$b^2 = c^2 - a^2$$

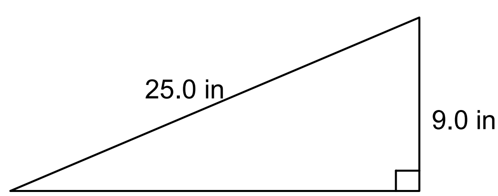
ASSIGNMENT 1 – PYTHAGOREAN THEOREM PRACTICE

1) Use the Pythagorean Theorem to calculate the unknown side length to one decimal place.

a)



b)



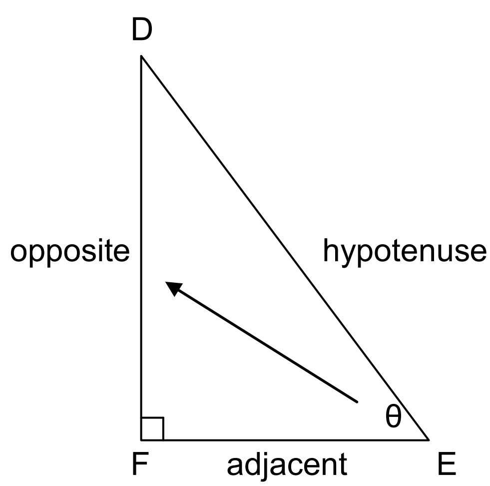
TRIGONOMETRY REVIEW

Trigonometry is one of the most important topics in mathematics. Trigonometry is used in many fields including engineering, architecture, surveying, aviation, navigation, carpentry, forestry, and computer graphics. Also, until satellites, the most accurate maps were constructed using trigonometry.

The word ***trigonometry*** means triangle measurements. It is necessary to finish our triangle facts here.

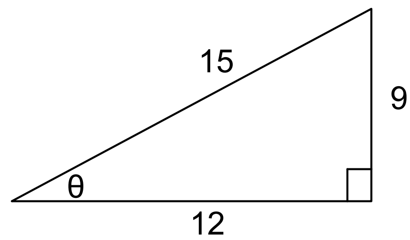
Fact 8: In trigonometry, the other two sides (or legs) of the triangle are referred to as the **opposite** and **adjacent** sides, depending on their relationship to the angle of interest in the triangle.

In this example, if we pick angle DEF – the angle labelled with the Greek letter θ – then we are able to distinguish the sides as illustrated in the diagram below.



The side that is opposite the angle of interest, in this case θ, is called the **opposite** side. The side that is nearest to angle θ and makes up part of the angle is called the **adjacent** side. To help you, remember that adjacent means beside. Although the hypotenuse occupies one of the two adjacent positions, it is **never** called the adjacent side. It simply remains the hypotenuse. This is why it is identified first. It is recommended to label the side in the order hypotenuse, opposite, and finally adjacent. You may use initials for these side, h, o, and a, but always use lower case letters to avoid mixing up the labelling with a vertex.

<u>Example 1</u>: Using the triangle below, answer the questions.



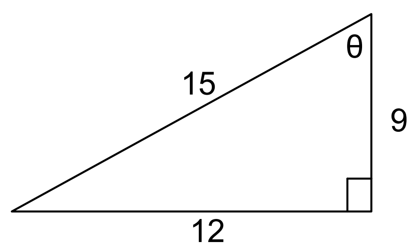
1) What is the hypotenuse? ___________
2) What is the opposite side to θ? ___________
3) What is the adjacent side to θ? ___________

<u>Solution</u>:

1) What is the hypotenuse? 15
2) What is the opposite side to θ? 9
3) What is the adjacent side to θ? 12

This example uses the same triangle as in Example 1; however, this time, the <u>*other*</u> acute angle is labelled as θ. This is done to show that the opposite and adjacent sides switch when the other angle is the angle of interest. The hypotenuse <u>**always**</u> stays the same.

<u>Example 2</u>: Using the triangle below, answer the questions.



1) What is the hypotenuse? ___________
2) What is the opposite side to θ? ___________
3) What is the adjacent side to θ? ___________

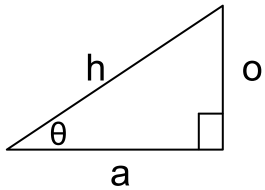
<u>Solution</u>:

1) What is the hypotenuse? 15
2) What is the opposite side to θ? 12
3) What is the adjacent side to θ? 9

ASSIGNMENT 2 – TRIGONOMETRY

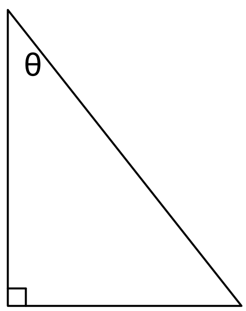
For each of the right triangles below, mark the hypotenuse, and the sides that are opposite and adjacent sides to θ as shown in the example.

Example:

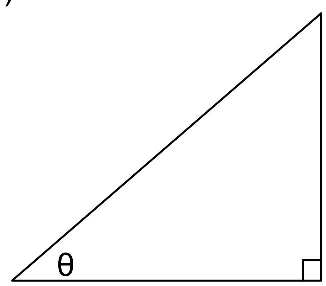


h = hypotenuse
o = opposite
a = adjacent

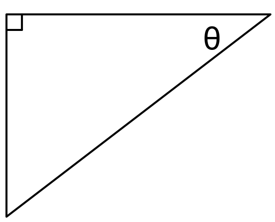
1)



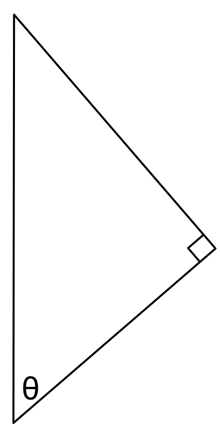
2)



3)



4)



TRIGONOMETRIC RATIOS

In the previous unit about similar figures, you learned that the ratios of corresponding sides of similar triangles are equal. When the angles of different triangles are the same, the ratio of the sides within the triangle will always be the same. They depend only on the measure of the angle of interest, not the size of the triangle. These ratios are the trigonometric ratios.

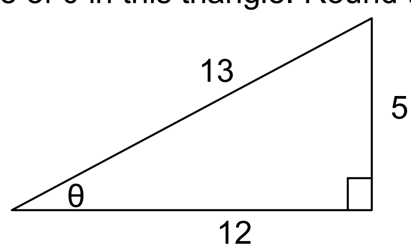
There are three trigonometric ratios we are concerned with: sine, cosine, and tangent.

THE SINE RATIO

The *sine of angle θ* means the ratio of the length of opposite side to the length of the hypotenuse. It is abbreviated as **sin θ** but read as sine θ. It is written like this:

$$\sin\theta = \frac{opposite}{hypotenuse} \quad \text{or } \sin\theta = \frac{o}{h}$$

Example 1: Find the sine of θ in this triangle. Round to 4 decimal places.



Solution:

The opposite side is 5 and the hypotenuse is 13. So

$$\sin\theta = \frac{o}{h} = \frac{5}{13} = 0.3846 \qquad \text{So } \sin\theta = 0.3846$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following sine ratios. Round to 4 decimal places.

a) $\sin 15^0$ b) $\sin 67^0$ c) $\sin 42^0$

******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) ******

Solution: Type “sin” followed by the angle, and then “=” to solve

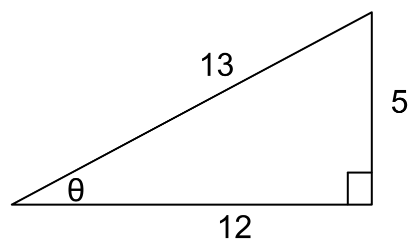
a) $\sin 15^0 = 0.2588$ b) $\sin 67^0 = 0.9205$ c) $\sin 42^0 = 0.6691$

THE COSINE RATIO

The *cosine of angle θ* means the ratio of the adjacent side to the hypotenuse. It is abbreviated as **cos θ** but read as cosine θ. It is written like this:

$$\cos\theta = \frac{adjacent}{hypotenuse} \quad \text{or} \quad \cos\theta = \frac{a}{h}$$

Example 1: Find the cosine of θ in this triangle.



Solution:

The adjacent side is 12 and the hypotenuse is 13. So

$$\cos\theta = \frac{a}{h} = \frac{12}{13} = 0.9231$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following cosine ratios. Round to 4 decimal places.

a) $\cos 15^0$ b) $\cos 67^0$ c) $\cos 42^0$

******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) ******

Solution: Type “cos” followed by the angle, and then “=” to solve

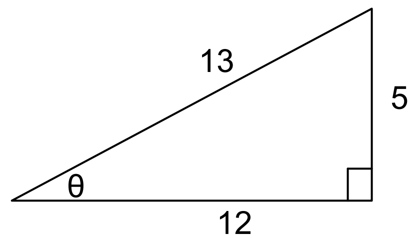
a) $\cos 15^0 = 0.9659$ b) $\cos 67^0 = 0.3907$ c) $\cos 42^0 = 0.7431$

THE TANGENT RATIO

The *tangent of angle θ* means the ratio of the opposite side to the adjacent side. It is abbreviated as **tan θ** but read as tangent θ. It is written like this:

$$\tan\theta = \frac{opposite}{adjacent} \quad \text{or } \tan\theta = \frac{o}{a}$$

Example 1: Find the tangent of θ in this triangle.



Solution:

The opposite side is 5 and the adjacent side is 12. So

$$\tan\theta = \frac{o}{a} = \frac{5}{12} = 0.4167$$

Note: Rounding to 4 decimal places is standard when calculating trigonometric ratios.

Example 2: Use your calculator to determine the following tangent ratios. Round to 4 decimal places.

a) $\tan 15^0$ b) $\tan 67^0$ c) $\tan 42^0$

******* REMEMBER TO SET YOUR CALCULATOR ON DEGREES (DEG) ******

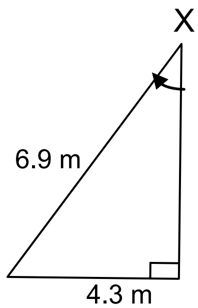
Solution: Type "tan" followed by the angle, and then "=" to solve

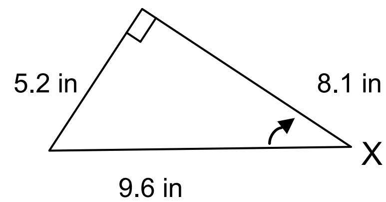
a) $\tan 15^0 = 0.2679$ b) $\tan 67^0 = 2.3559$ c) $\tan 42^0 = 0.9004$

ASSIGNMENT 3 – THE TRIGONOMETRIC RATIOS

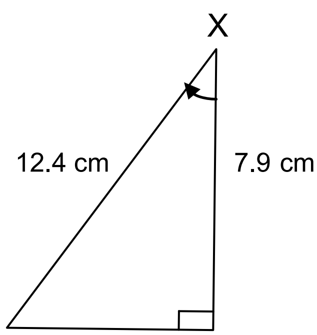
1)

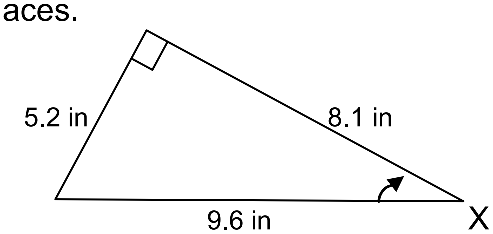
a) Calculate the value of **sin X** to two decimal places.



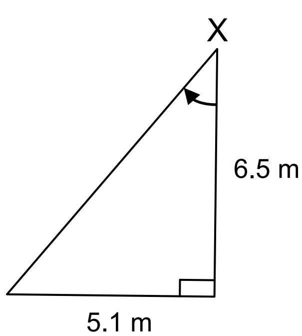


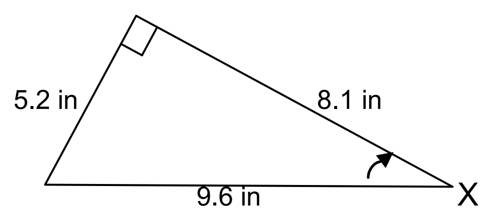
b) Calculate the value of **cos X** to two decimal places.





c) Calculate the value of **tan X** to two decimal places.





2) Use your calculator to determine the value of each of the following sine ratios to four decimal places.

a) $\sin 10^0$ = _______________ b) $\sin 48^0$ = _______________

c) $\sin 77^0$ = _______________ d) $\sin 85^0$ = _______________

e) $\cos 10^0$ = _______________ f) $\cos 48^0$ = _______________

g) $\cos 77^0$ = _______________ h) $\cos 85^0$ = _______________

i) $\tan 10^0$ = _______________ j) $\tan 48^0$ = _______________

k) $\tan 77^0$ = _______________ l) $\tan 85^0$ = _______________

3) There are two special sine ratios. Calculate the following and suggest why the values are what the results give you.

a) $\sin 0^0$ = _______________ b) $\sin 90^0$ = _______________

4) There are two special cosine ratios. Calculate the following and suggest why the values are what the results give you.

a) $\cos 0^0$ = _______________ b) $\cos 90^0$ = _______________

5) There are some special tangent ratios. Calculate the following and suggest why the values are what the results give you.

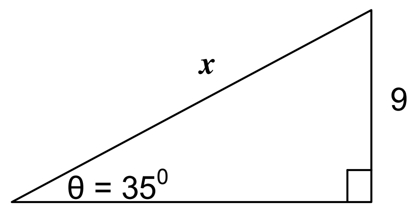
a) $\tan 0^0$ = _______________ b) $\tan 45^0$ = _______________

c) $\tan 89^0$ = _______________ d) $\tan 90^0$ = _______________

USING THE SINE RATIO

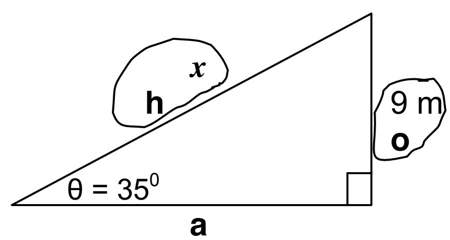
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The sine ratio can be used to find missing parts of a right triangle.

Example 1: Use the sine ratio to find the x in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$\sin \theta = \frac{o}{h}$

$\sin 35^0 = \frac{9}{x}$

Step 5: Solve using the process Cross Multiply and Divide.

Since $\sin 35^0 = \frac{\sin 35}{1}$, then $\sin 35^0 = \frac{9}{x}$ becomes $\frac{\sin 35}{1} = \frac{9}{x}$

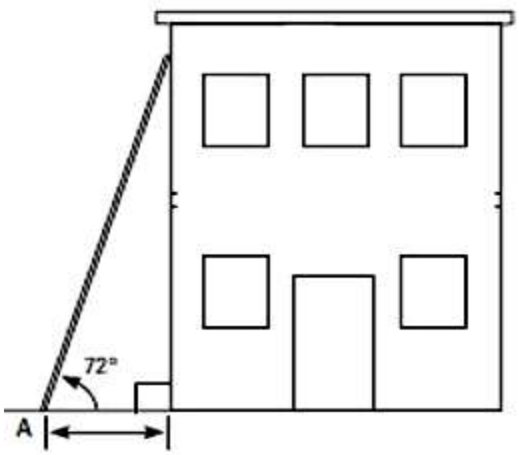
$x = 9 \times 1 \div \sin 35^0$

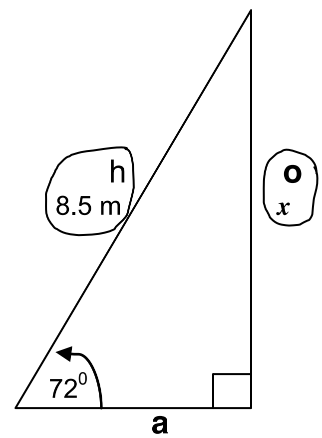
$= 15.7$ m

Example 2: A ladder 8.5 m long makes an angle of 72^0 with the ground. How far up the side of a building will the ladder reach?

Solution:

Sketch a diagram and place the information from the question on this diagram. Remember that there will always be a right triangle in your diagram. It is often helpful to draw that triangle and copy the key information from the sketch.





Step 1: Label the sides of the triangle with **h**, **o** and **a**
See above right.

Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **h** are being used, the correct ratio is **sin θ**

Step 4: Substitute the correct values into the correct ratio.

$\sin \theta = \frac{o}{h}$

$\sin 72^0 = \frac{x}{8.5}$

Step 5: Solve using the process Cross Multiply and Divide.

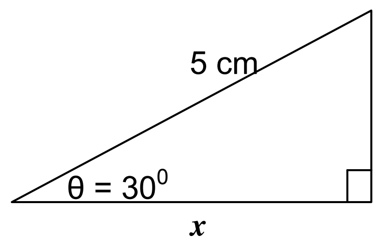
Since $\sin 72^0 = \frac{\sin 72}{1}$, then $\sin 72^0 = \frac{x}{8.5}$ becomes $\frac{\sin 72}{1} = \frac{x}{8.5}$

$x = \sin 72^0 \times 8.5 \div 1$
$= 8.1$ m

USING THE COSINE RATIO

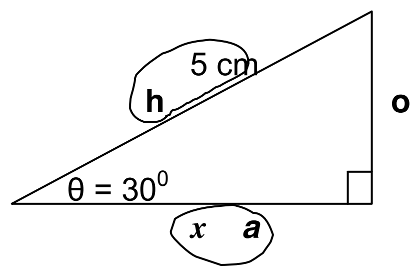
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The cosine ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the ***x*** in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **a** and **h** are being used, the correct ratio is **cos θ**

Step 4: Write down the chosen ratio and substitute the correct values into the correct ratio.

$\cos\theta = \frac{a}{h}$

$\cos 30^0 = \frac{x}{5}$

Step 5: Solve using the process Cross Multiply and Divide.

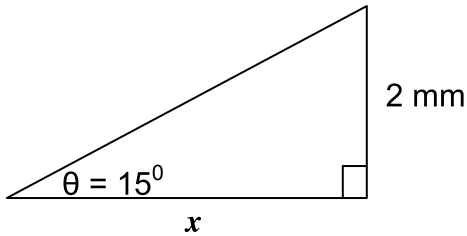
Since $\cos 30^0 = \frac{\cos 30}{1}$, then $\cos 30^0 = \frac{x}{5}$ becomes $\frac{\cos 30}{1} = \frac{x}{5}$

$x = \cos 30^0 \times 5 \div 1$
$= 4.3$ cm

USING THE TANGENT RATIO

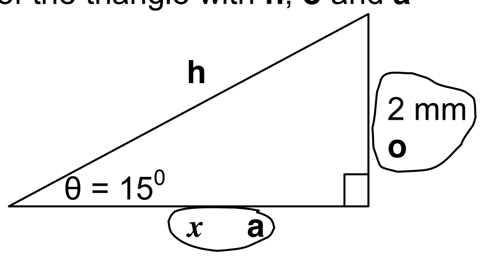
Whenever one side and one angle of a right triangle are already known, the remaining sides can be found using the trigonometric ratios. The tangent ratio can be used to find missing parts of a right triangle.

Example 1: Use the correct trig ratio to find the ***x*** in the triangle below.



Solution:

Step 1: Label the sides of the triangle with **h**, **o** and **a**



Step 2: Circle the number with the side it represents and the unknown (x) with the side it represents.

Step 3: Identify the ratio required to solve for x

Since **o** and **a** are being used, the correct ratio is **tan θ**

Step 4: Substitute the correct values into the correct ratio.

$$\tan \theta = \frac{o}{a}$$

$$\tan 15 = \frac{2}{x}$$

Step 5: Solve using the process Cross Multiply and Divide.

Since $\tan 15^0 = \frac{\tan 15}{1}$, then $\tan 15^0 = \frac{2}{x}$ becomes $\frac{\tan 15}{1} = \frac{2}{x}$

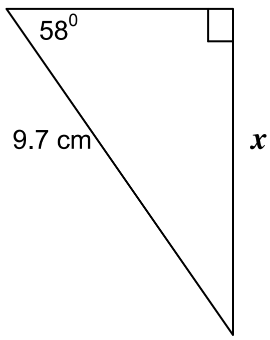
$x = 2 \times 1 \div \tan 15^0$

$= 7.5$ mm

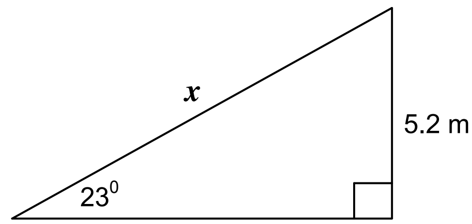
ASSIGNMENT 4 – FINDING SIDES IN RIGHT TRIANGLES

1) Calculate the length of the side indicated in the following diagrams. Round to **one** decimal place.

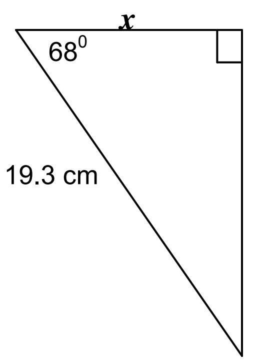
a)



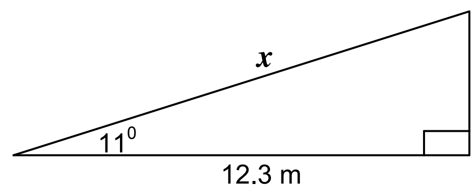
b)



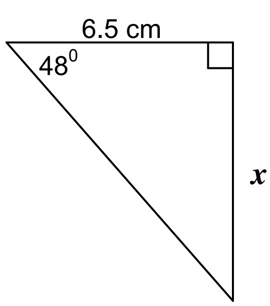
c)



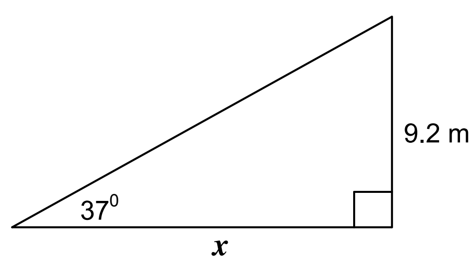
d)



e)



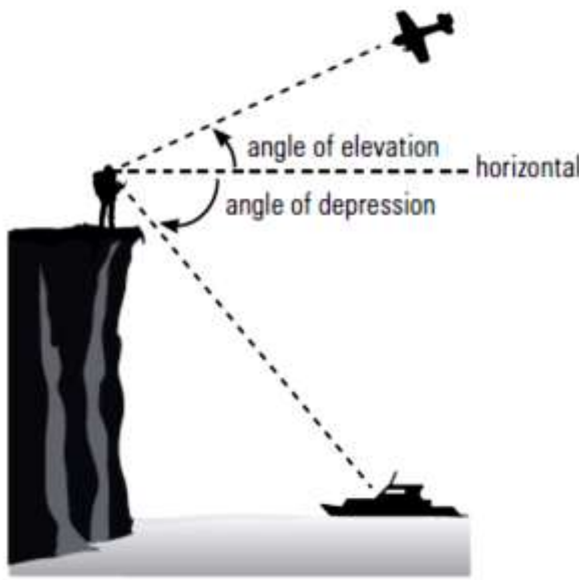
f)



ASK YOUR TEACHER FOR THE UNIT QUIZ 1.

ANGLE OF ELEVATION AND DEPRESSION

When you look up at an airplane flying overhead for example, the angle between the horizontal and your line of sight is called the **angle of elevation**.

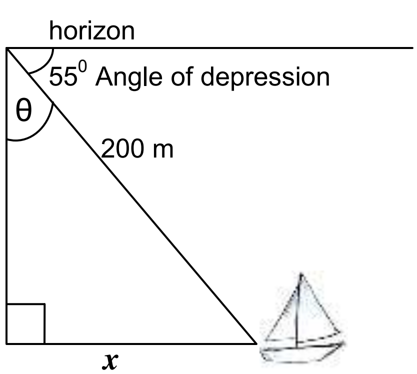


When you look down from a cliff to a boat passing by, the angle between the horizontal and your line of sight is called the **angle of depression**.

When you are given the angle of depression, it is important to carefully use this angle in your triangle.

Example 1: You are standing at the top of a cliff. You spot a boat 200 m away at an angle of depression of 55^0 to the horizon. How far is the boat from the coast? Draw a diagram to illustrate this situation.

Solution: Draw a diagram, label it with the information, and then solve the triangle.



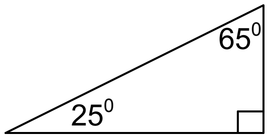
The angle inside the triangle is the complement to the angle of depression. To find that angle, do the following:

$\theta = 90^0 - 55^0$

$\theta = 35^0$

ASSIGNMENT 5 – ANGLE OF ELEVATION AND DEPRESSION

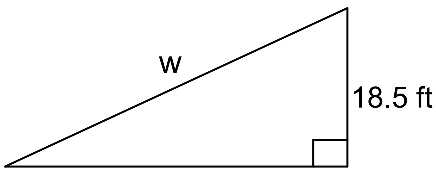
1) In the triangle below, what is the measure of the angle of elevation?



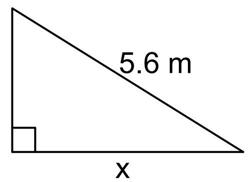
Measure ________________

2) Write the angle of elevation in each diagram. Then find the length of the unknown side, to one decimal place.

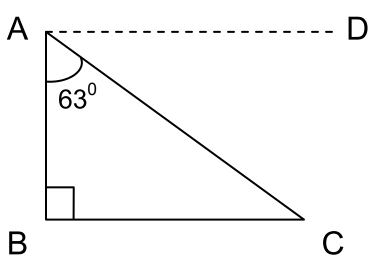
a) Angle of elevation = 43^0



b) Angle of elevation = 21^0



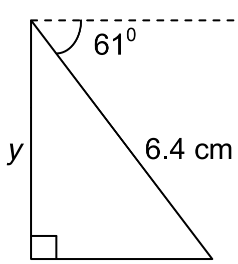
3) In the diagram below, name the angle of depression. What is the measure of this angle?

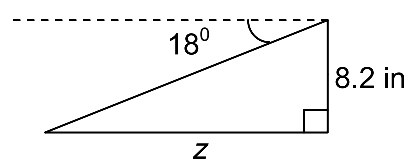


Name ___________________

Measure ________________

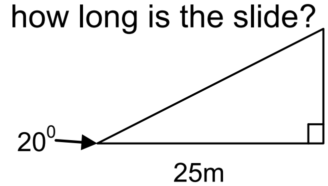
4) Find the length of the unknown side, to one decimal place.



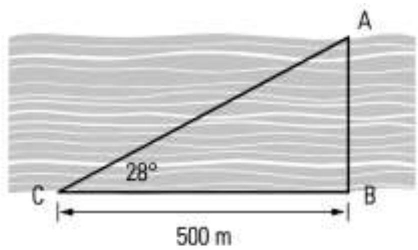


ASSIGNMENT 6 – WORD PROBLEMS

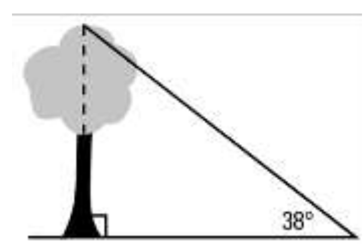
1) A child's slide rises to a platform at the top. If the angle of elevation of the slide is 20^0, and the horizontal distance that the slide covers is 25 m long, how long is the slide?



2) A surveyor must determine the distance AB across a river. If he knows the information in the diagram below, how wide is the river?



3) A tree is measured to be 12.4 m tall. If a man views the top of the tree at an angle of elevation of 38^0, how far away from the tree is he standing?



4) A ladder is placed against the side of the house. If the base is 41 feet away from the house, and the angle of elevation between the ladder and the ground is 70^0, how long is the ladder?

FINDING ANGLES IN RIGHT TRIANGLES

So far in this unit, you have used the trigonometric ratios to find the length of a side. But if you know the trigonometric ratio, you can calculate the size of the angle. This requires and "inverse" operation. You can use your calculator to find the opposite of the usual ratio provided you can calculate the ratio. To do this you need 2 sides in the triangle. You can think of the inverse in terms of something simpler: addition is the opposite or inverse of subtraction. In the same way, trig functions have an inverse.

To calculate the inverse, you usually use a 2nd function and the sin/cos/tan buttons on your calculator in sequence. If you look at your calculator just above the sin/cos/tan buttons, you should see the following: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$. These are the inverse functions. If you use these buttons, you will be able to turn a ratio into an angle.

Example 1: Calculate each angle to the nearest whole degree.

a) $\sin X = 0.2546$

b) $\cos Y = 0.1598$

c) $\tan Z = 3.2785$

Solution: Use the appropriate inverse function on your calculator.

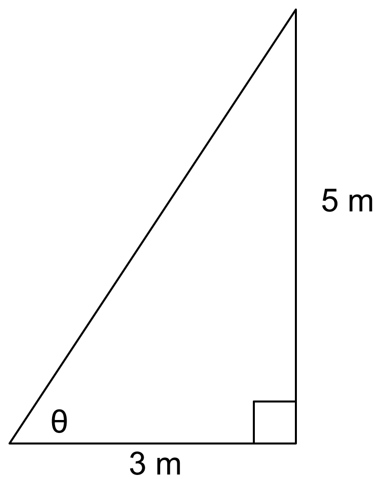
NOTE: Every calculator is different in how the buttons are keyed in order to achieve the desired outcome. Most calculators will need to key "**2ndF sin**" or "**Shift sin**" in order to get $\sin^{-1}$ displayed. Then key in the value with or without brackets as necessary.

a) $\sin X = 0.2546$
$X = \sin^{-1}(0.2546)$
$X = 14.74988^0$ Angle X is 15^0.

b) $\cos Y = 0.1598$
$Y = \cos^{-1}(0.1598)$
$Y = 80.8047^0$ Angle Y is 81^0.

c) $\tan Z = 3.2785$
$Z = \tan^{-1}(3.2785)$
$Z = 73.03737^0$ Angle Z is 73^0.

Example 2: Determine the angle θ in the following triangle.

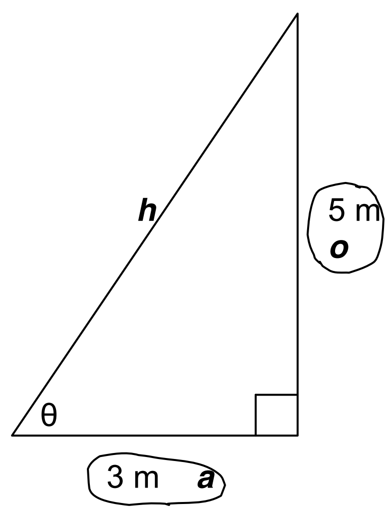


Solution:

1) h, o, a the triangle

2) Circle the letters with their partner numbers

3) Choose the appropriate trig ratio. In this case, it is tangent.



4) Write down the ratio and fill it in.

$$\tan \theta = \frac{o}{a}$$

$$\tan \theta = \frac{5}{3}$$

5) Divide the numerator (top number) by the denominator (bottom number) in the fraction to get a decimal number.

$$\tan \theta = 1.66666$$

6) Use the inverse function to solve for θ.

$$\theta = \tan^{-1}(1.66666)$$
$$\theta = 59.0352^{0}$$

Angle θ is approximately 59^{0}.

ASSIGNMENT 7 – FINDING ANGLES IN RIGHT TRIANGLES

1) Calculate the following angles to the nearest whole degree.

a) sin D = 0.5491

b) cos F = 0.8964

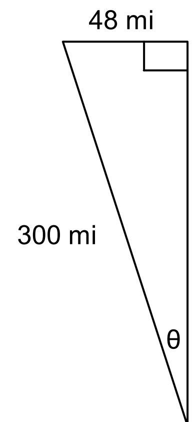
c) tan G = 2.3548

d) sin P = 0.9998

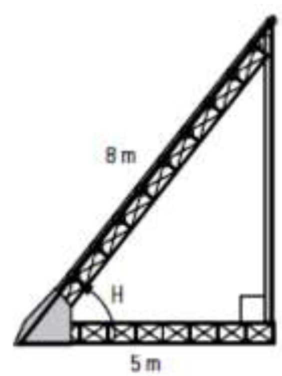
e) cos Q = 0.3097

f) tan R = 0.4663

2) After an hour of flying, a jet has travelled 300 miles, but gone off course 48 miles west of its planned flight path. What angle, θ, is the jet off course?



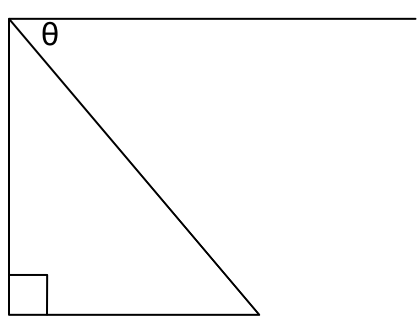
3) At what angle to the ground is an 8 m long conveyor belt if it is fastened 5 m from the base of the loading ramp?



4) If a boat is 150 m from the base of a 90 m cliff, what is the angle of elevation from the boat to the top of the cliff?

5) A statue of Smokey the Bear is found in Revelstoke, BC. At a distance of 6.3 m from the base, the angle of elevation to the top is 55^0. How tall is the statue?

6) What is the angle of depression, θ, from the top of a 65 m cliff to an object 48 m from its base?

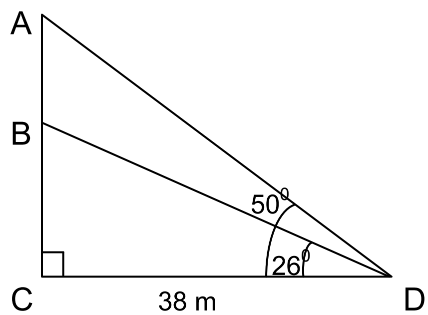


<u>ASK YOUR TEACHER FOR THE UNIT QUIZ 2.</u>

SOLVING COMPLEX PROBLEMS

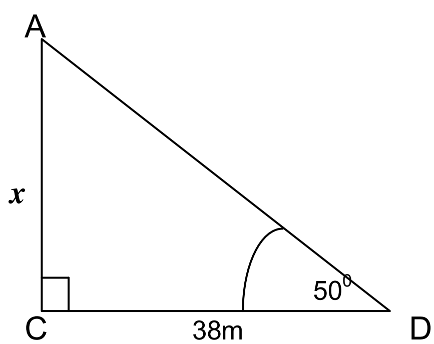
In some circumstances, you will have two or more triangles together in one diagram, and you will need to complete several steps in order to find the answer for the angle or the side you are specifically asked for. These multi-step problems are no harder than a single triangle problem as long as you follow through with the method you have been taught.

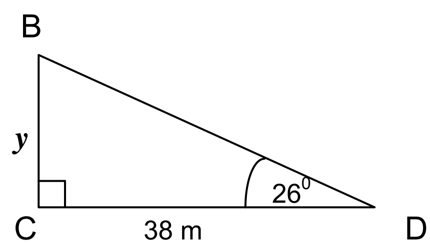
Example: In the following diagram, find the length of AB.



NOTE: The way the two (or three) triangles are arranged will not always be the same as shown in this example. It is helpful to draw the individual triangles and work with them separately.

Solution: Find the lengths of AC and BC using the appropriate trig ratio. Then subtract to find AB.





$\tan\theta = \frac{o}{a}$

$\tan 50^0 = \frac{x}{38}$

$x = \tan 50^0 \times 38$

$x = 45.3$ m

$\tan\theta = \frac{o}{a}$

$\tan 26^0 = \frac{y}{38}$

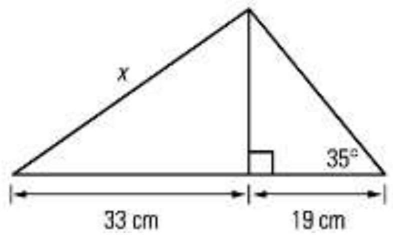
$y = \tan 26^0 \times 38$

$y = 18.5$ m

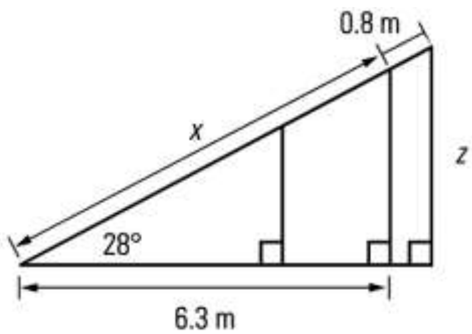
So AB = AC – BC = 45.3 – 18.5 = 26.8 m

ASSIGNMENT 8 – WORKING WITH TWO TRIANGLES

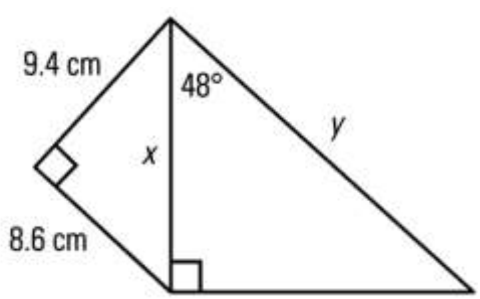
1) What is the length of *x* in the diagram below?



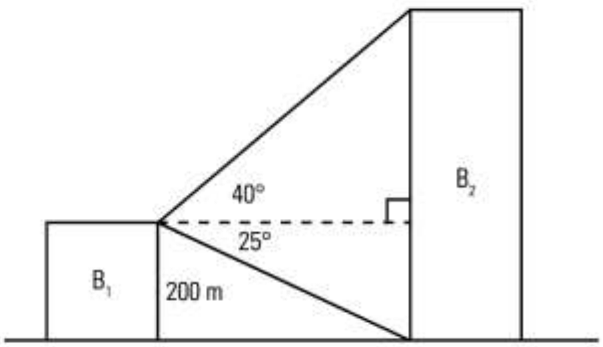
2) Find the lengths of *x* and *z* below.



3) Find the lengths of *x* and *y* below. Hint: use Pythagorean Theorem to find *x*.



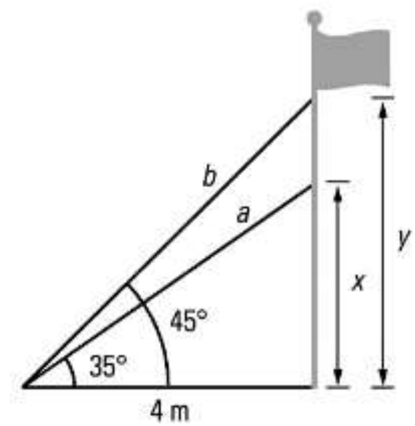
4) From the top of 200 m tall office building (B_1), the angle of elevation to the top of another building (B_2) is 40^0. The angle of depression to the bottom of that building is 25^0. How tall is that second building (B_2)?



5) A flagpole is supported by two guy wires, each attached to the same peg in the ground that is 4 m from the base of the flagpole. The guy wires have angles of elevation of 35^0 and 45^0 as shown below.

a) How long is each guy wire, lengths *a* and *b*?

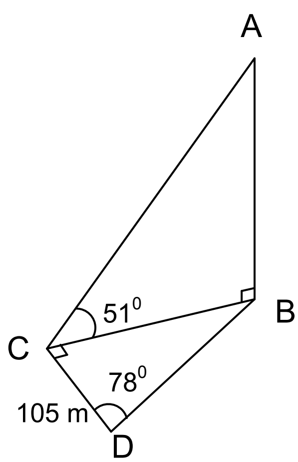
b) How much higher up the flagpole is the top guy wire attached?



SOLVING COMPLEX PROBLEMS IN THE REAL WORLD

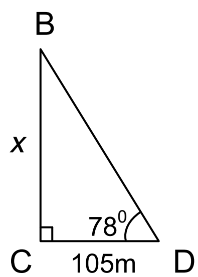
In some situations, you will need to work with triangles that are at an angle to each other. Also, some situations will involve triangles that share a common edge but in each triangle this edge will represent a different dimension. It is sometimes hard to visualize these diagrams as they are trying to represent three dimensional images on a two dimensional paper. If you are having difficulties, draw the triangles separately and work that way. Remember, we are **always** using right triangles in these problems.

Example: Calculate the height of a cliff, AB below, given the information on the diagram.



Solution: Use ΔBCD to find side BC, and then use that length in ΔABC to find AB

Step 1: ΔBCD



$\tan \theta = \frac{o}{a}$

$\tan 78^0 = \frac{x}{105}$

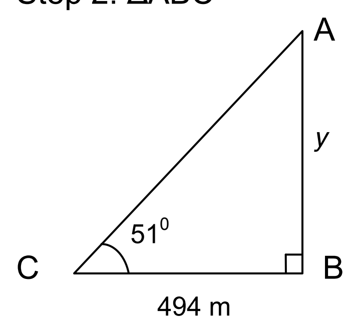
$x = \tan 78^0 \times 105$

$x = 493.986\ .\ .\ .$ m

$x = 494$ m

Now use this length in the second triangle.

Step 2: ΔABC



$\tan \theta = \frac{o}{a}$

$\tan 51^0 = \frac{y}{494}$

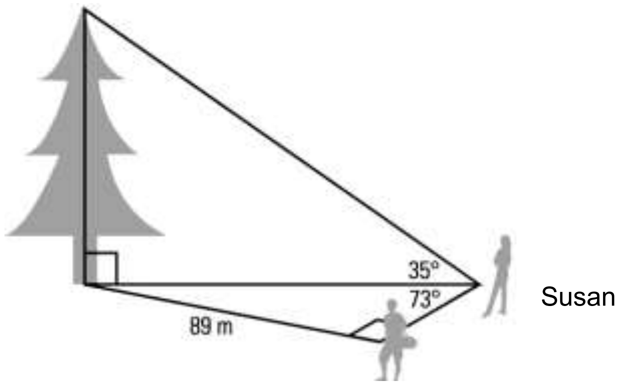
$y = \tan 51^0 \times 494$

$y = 610.0$ m

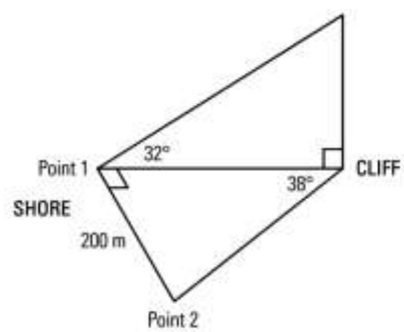
The height of the cliff, AB = 610 m

ASSIGNMENT 9 – WORKING WITH TRIANGLES IN 3-D

1) Susan and Marc spot a bird's nest at the top of a tree. Marc is 89 m from the tree. The angle between Susan's line of sight and Marc's line of sight is 73^0. If the angle of elevation from Susan to the top of the tree is 35^0, what is the height of the nest in the tree – how tall is the tree?



2) You need to calculate the height of a cliff that drops vertically into a river. Use the information in the diagram to calculate the height of the cliff.



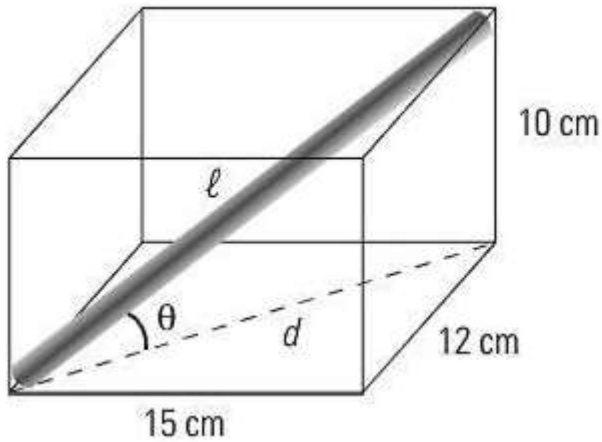
3) An airplane is flying 100 km north and 185 km west of an airport. It is flying at a height of 7 km.

a) Draw a diagram to show this problem. There should be 2 right triangles in your diagram. Label the vertices of each triangle with letters.

b) Using your diagram to help, calculate the straight-line distance from the plane to the airport.

c) What is the angle of elevation of the plane from the airport?

4) A box (shown below) is 10 cm by 12 cm by 15 cm. Length *d* is the diagonal along the bottom of the box.



a) What is the length of the longest rod that can be carried in this box?

b) What angle, θ, does it make with the bottom of the box?